EG UK Computer Graphics & Visual Computing (2014), pp. 1–2
Rita Borgo, Wen Tang (Editors)

# Simplifying Multivariate Topology (Extended Abstract)

Amit Chattopadhyay[1], Hamish Carr[1], David Duke[1] and Zhao Geng[1]

[1]School of Computing, University of Leeds, UK

**Abstract**

*Topological simplification is effective for analysis and visualisation of scalar data. However, topological simplification of multivariate data is less well developed. We describe multi-field topology simplification strategy based on the Reeb Space. We generalise critical nodes in the Reeb Graph to the Jacobi Structure of the Reeb Space that divides it into regions. The dual graph for these regions, the Reeb Skeleton, has properties similar to the Reeb graph, and can be simplified using importance measures based on generalising persistence to Measure Persistence.*

Categories and Subject Descriptors (according to ACM CCS): I.3.6 [Computer Graphics]: Methodology and Techniques—Graphics data structures and data types

## 1. Context

Scientific data is complex in nature and difficult to visualise. Topological tools have therefore become important in scientific visualisation, especially for scalar fields [CSvdP10, TP12]. Multivariate topology has been based on Jacobi sets - a generalisation of critical points to multifields [EH04, BBD*07]. Although the Reeb graph has been generalised to the Reeb space [EHP08], tools for simplification have only recently become feasible using a quantised approximation called the Joint Contour Net (JCN) [CD13]. We report on recent results in multivariate simplification.

**Fiber Analysis and Jacobi Sets.** Where scalar analysis depends on Morse theory, multivariate analysis depends on fiber topology. A *fiber* or a level set of a multi-field is the inverse image of a range value. Fibers at which topological events occur are called singular fibers, while other fibers are called regular fibers. The *Jacobi Set* of a multi-field is the set of critical points in the the domain [EH04].

**Reeb Space and JCN.** As the Reeb graph of a scalar field, the *Reeb Space* captures the fiber topology of a multi-field. Each connected component of a fiber is called a joint contour and corresponds to a point in the Reeb space [EHP08]. Generically, for a multi-field $f : \mathbb{R}^d \to \mathbb{R}^r$, with $r \leq d$, the Reeb space consists of a collection of $r$-manifolds glued together in complicated ways. The JCN is a multiresolution data-structure that approximates the Reeb space of a piecewise-linear multi-field [CD13]. We implement our Reeb space simplification strategy in the JCN framework.

## 2. Our Method

Our Reeb space simplification method is based on reducing the Reeb space into a graph skeleton and then applying a similar set of strategies as in contour tree or Reeb graph simplification. We outline the main steps of our method.

**A. Extracting Jacobi Structures in Reeb Spaces.** Each point in a Jacobi Set maps a singular fiber to the Reeb Space, not necessarily uniquely. Instead, the Reeb Space is partitioned by the projection of the Jacobi Set, i.e. the *Jacobi Structure* [CCDG14]. As a result, the Jacobi Structure is equivalent to the critical nodes in a Reeb Graph, and captures the relationships between features (i.e. regions). We extract the Jacobi Structure of a given JCN by building a Multi-Dimensional Reeb Graph (MDRG) [CCDG14].

**B. Finding Regular and Singular Components.** In scalar fields, critical points in the Reeb graph correspond to separating contours in the domain, while edges correspond to features. In the Reeb space, these boundaries are $r-1$ manifolds of the Jacobi structure separating $r$-manifold features. Each $r$-manifold component is called a regular component and the $r-1$ manifold components of the Jacobi structure, partitioned based on their adjacency with the regular components, are called the singular components.

**C. Reeb Skeleton.** Once the role of the $r-1$ manifold components of the Jacobi structure is recognised, it is possible to perform a further reduction from the Reeb Space. To do so, we represent both these $r-1$ manifold components and the

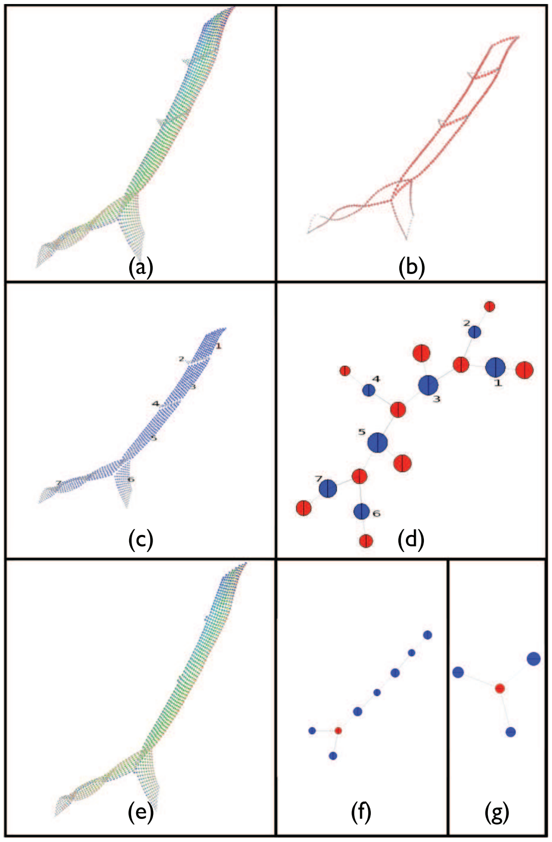


Figure 1: (**Simplification Demo**) (a) Original JCN/Reeb Space of bivariate field (Parab, Height) (b) Jacobi Structure, (c) Regular components (d) Reeb Skeleton ('blue' corresponds to regular components and 'red' corresponds to adjacent singular components) (e) Simplified JCN (f)-(g) Simplified Reeb Skeleton using persistent metric.

$r$ manifold regular components as points, and add edges representing their connectivity: in short, we can build the dual graph of these components of the Reeb space. This has the merit of further reducing the Reeb space from an $r$- manifold structure to a fundamentally 1-manifold structure which is easier to represent, to reason about and to visualise. We refer to this as the *Reeb Skeleton*.

**D. Simplification Metrics–Measure Persistence.** In simplifying the contour tree, Reeb Graph and Morse-Smale Complex, simplification can be defined by cancelling pairs of critical points according to an ordering given by a *filtration* - i.e. a sequence by which simplices are added to a complex. In multifields, the persistence of a feature gives rise to tuples rather than a single value [CZ09], which does not naturally give rise to a total ordering of the features. We define the *measure persistence* of a regular component in the Reeb space to be the geometric measure of its projection into the range of $f$ which induces a canonical ordering that is invariant under linear range transformations.

**Geometric Measures.** We also compute other geometric measures of the regular components, either in the domain, in the range, or in some combination of the two, using geometric measure theory, similar as with the contour tree [CSvdP10].

## 3. Results

The main results of this paper are as follows: (1) Introduces the *Jacobi Structure* in the Reeb Space that decomposes the Reeb Space into manifold components equivalent to edges and vertices in the Reeb Graph, (2) Generalises scalar persistence to *measure persistence*, inducing a total ordering of regular components of the Reeb Space, (3) Describes an algorithm that extracts the Jacobi Structure of a Joint Contour Net hierarchically using a Multi-dimensional Reeb Graph, then reduces it further to a Reeb Skeleton, and (4) Simplifies the Reeb Space and Reeb Skeleton using approximated measure persistence and geometric measures.

## 4. Novelty

In this paper, we have shown that the Reeb space can be decomposed and simplified by simple rules that generalise approaches that are effective for scalar fields, and that the Jacobi Structure that characterises the Reeb space is richer than the Jacobi Set. We have also shown that persistence can be generalised to *measure persistence*, providing a rigorous foundation for simplification.

## References

[BBD*07] BREMER P.-T., BRINGA E. M., DUCHAINEAU M., LANEY D., MASCARENHAS A., PASCUCCI V.: Topological feature extraction and tracking. *Journal of Physics: Conference Series 78:012007* (2007). 1

[CCDG14] CHATTOPADHYAY A., CARR H., DUKE D., GENG Z.: Extracting jacobi structures in reeb spaces. In *The Eurographics Conference on Visualization, Eurovis 2014, 9-13 June, Swansea, UK, Short Paper* (2014). 1

[CD13] CARR H., DUKE D.: Joint Contour Nets. *IEEE Transactions on Visualization and Computer Graphics* (2013), xxx–xxx. accepted. 1

[CSvdP10] CARR H., SNOEYINK J., VAN DE PANNE M.: Flexible isosurfaces: Simplifying and displaying scalar topology using the contour tree. *Computational Geometry: Theory and Applications 43*, 1 (2010), 42–58. 1, 2

[CZ09] CARLSSON G., ZOMORODIAN A.: The Theory of Multidimensional Persistence. *Discrete and Computational Geometry - 23rd Annual Symposium on Computational Geometry 42*, 1 (2009), 71–93. 2

[EH04] EDELSBRUNNER H., HARER J.: Jacobi Sets of Multiple Morse Functions. *Foundations of Computational Mathematics* (2004), 37–57. Cambridge Univ. Press, 2004. 1

[EHP08] EDELSBRUNNER H., HARER J., PATEL A. K.: Reeb Spaces of Piecewise Linear Mappings. In *SoCG* (2008). 1

[TP12] TIERNY J., PASCUCCI V.: Generalized topological simplification of scalar fields on surfaces. *IEEE Trans. on Visualization and Computer Graphics 18*, 12 (2012), 2005–2013. 1